Deriving the Quadratic Formula

REASONS	STEPS
• Given a quadratic equation	$ax^2 + bx + c = 0$
• Isolate the constant, c	$ax^2 + bx = -c$
• Divide both sides by the leading coefficient, a. • Complete the square by taking half of the linear term (x-term) and square it. • Add the squared term to both sides.	$x^2 + \frac{b}{a}x = -\frac{c}{a}$ $\frac{b}{2a} \to \frac{b^2}{4a^2}$ $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$
• Simplify on the right-hand side; in this case, simplify by converting to a common denominator.	$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$
• Rewrite the left-hand side to a square of a binomial. • Simplifying on the right by adding the fractional terms.	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
• Take the square of both sides. Note: $\sqrt[n]{u^n} = \lvert u \rvert, when\ n\ is\ even$	$\left\lvert x + \frac{b}{2a} \right\rvert = \sqrt{\frac{b^2 - 4ac}{4a^2}}$
• Note: $\lvert u \rvert = \begin{cases} u \\ or \\ -u \end{cases}$	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$
• Isolate the x-variable. • Simplify the right side by converting to a common denominator.	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$